DPP - Daily Practice Problems

Chapter-wise Sheets

Date : ______ Start Time : ______ End Time : ______

PHYSICS

CP13

SYLLABUS : Oscillations

Max. Marks : 180 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

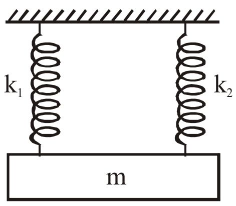
INSTRUCTIONS : This Daily Practice Problem Sheet contains 45 MCQs. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?

(a) aT/x (b) $aT + 2\pi v$

(c) aT/v (d) $a^2T^2 + 4\pi^2v^2$

2. A mass is suspended separately by two different springs in successive order, then time periods is t_1 and t_2 respectively. It is connected by both springs as shown in fig. then time period is t_0. The correct relation is



(a) $t_0^2 = t_1^2 + t_2^2$

(b) $t_0^{-2} = t_1^{-2} + t_2^{-2}$

(c) $t_0^{-1} = t_1^{-1} + t_2^{-1}$

(d) $t_0 = t_1 + t_2$

3. A rod of length ℓ is in motion such that its ends A and B are moving along x-axis and y-axis respectively. It is given that $\frac{d\theta}{dt} = 2$ rad/sec always. P is a fixed point on the rod. Let M be the projection of P on x-axis. For the time interval in which θ changes from 0 to $\frac{\pi}{2}$, the correct statement is

(a) The acceleration of M is always directed towards right

(b) M executes SHM

(c) M moves with constant speed

(d) M moves with constant acceleration

4. A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is

(a) $2\pi^2 ma^2\nu^2$ (b) $\pi^2 ma^2\nu^2$

(c) $\frac{1}{4}ma^2\nu^2$ (d) $4\pi^2 ma^2\nu^2$

5. A mass M attached to a spring oscillates with a period of 2s. If the mass is increased by 2 kg, then the period increases by 2s. Find the initial mass M assuming that Hooke's law is obeyed.

(a) $\frac{2}{3}$kg (b) $\frac{1}{3}$kg (c) $\frac{1}{2}$kg (d) 1 kg

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Space for Rough Work

6. The amplitude of a damped oscillator becomes $\left(\frac{1}{3}\right)^{rd}$ in 2 seconds. If its amplitude after 6 seconds is $\frac{1}{n}$ times the original amplitude, the value of n is
(a) 3^2 (b) 3^3 (c) $\sqrt[3]{3}$ (d) 2^3

7. Assume the earth to be perfect sphere of uniform density. If a body is dropped at one end of a tunnel dug along a diameter of the earth (remember that inside the tunnel the force on the body is – k times the displacement from the centre, k being a constant), it (body) will
(a) reach the earth's centre and stay there
(b) go through the tunnel and comes out at the other end
(c) oscillate simple harmonically in the tunnel
(d) stay somewhere between the earth's centre and one of the ends of tunnel.

8. A particle undergoes simple harmonic motion having time period T. The time taken in 3/8th oscillation is
(a) $\frac{3}{8}T$ (b) $\frac{5}{8}T$ (c) $\frac{5}{12}T$ (d) $\frac{7}{12}T$

9. A particle is executing simple harmonic motion with amplitude A. When the ratio of its kinetic energy to the potential energy is $\frac{1}{4}$, its displacement from its mean position is
(a) $\frac{2}{\sqrt{5}}A$ (b) $\frac{\sqrt{3}}{2}A$ (c) $\frac{3}{4}A$ (d) $\frac{1}{4}A$

10. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is
(a) 11% (b) 21% (c) 42% (d) 10%

11. The time period of a mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be
(a) $2T$ (b) $\frac{T}{4}$ (c) 2 (d) $\frac{T}{2}$

12. Two simple harmonic motions act on a particle. These harmonic motions are $x = A\cos(\omega t + \delta)$, $y = A\cos(\omega t + \alpha)$ when $\delta = \alpha + \frac{\pi}{2}$, the resulting motion is
(a) a circle and the actual motion is clockwise
(b) an ellipse and the actual motion is counterclockwise
(c) an elllipse and the actual motion is clockwise
(d) a circle and the actual motion is counter clockwise

13. A point mass oscillates along the x-axis according to the law $x = x_0\cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t - \delta)$, then
(a) $A = x_0\omega^2, \delta = 3\pi/4$ (b) $A = x_0, \delta = -\pi/4$
(c) $A = x_0\omega^2, \delta = \pi/4$ (d) $A = x_0\omega^2, \delta = -\pi/4$

14. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $\frac{5T}{3}$. Then the ratio of $\frac{m}{M}$ is
(a) $\frac{3}{5}$ (b) $\frac{25}{9}$ (c) $\frac{16}{9}$ (d) $\frac{5}{3}$

15. A body oscillates with a simple harmonic motion having amplitude 0.05 m. At a certain instant of time, its displacement is 0.01 m and acceleration is 1.0 m/s^2. The period of oscillation is
(a) 0.1 s (b) 0.2 s (c) $\frac{\pi}{10}$s (d) $\frac{\pi}{5}$s

16. The particle executing simple harmonic motion has a kinetic energy $K_0\cos^2\omega t$. The maximum values of the potential energy and the total energy are respectively
(a) $K_0/2$ and K_0 (b) K_0 and $2K_0$
(c) K_0 and K_0 (d) 0 and $2K_0$

17. A simple pendulum attached to the ceiling of a stationary lift has a time period T. The distance y covered by the lift moving upwards varies with time t as $y = t^2$ where y is in metres and t in seconds. If g = 10 m/s^2, the time period of pendulum will be
(a) $\sqrt{\frac{4}{5}}T$ (b) $\sqrt{\frac{5}{6}}T$ (c) $\sqrt{\frac{5}{4}}T$ (d) $\sqrt{\frac{6}{5}}T$

18. A particle moves with simple harmonic motion in a straight line. In first τs, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then:
(a) amplitude of motion is 3a
(b) time period of oscillations is 8τ
(c) amplitude of motion is 4a
(d) time period of oscillations is 6τ

19. Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1\cos\pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is
(a) $\frac{\pi}{3}$ (b) $\frac{-\pi}{6}$ (c) $\frac{\pi}{6}$ (d) $\frac{-\pi}{3}$

20. Masses M_A and M_B hanging from the ends of strings of lengths L_A and L_B are executing simple harmonic motions. If their frequencies are $f_A = 2f_B$, then
(a) $L_A = 2L_B$ and $M_A = M_B/2$
(b) $L_A = 4L_B$ regardless of masses
(c) $L_A = L_B/4$ regardless of masses
(d) $L_A = 2L_B$ and $M_A = 2M_B$

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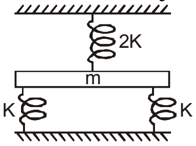
Space for Rough Work

21. In damped oscillations, the amplitude of oscillations is reduced to one-third of its inital value a_0 at the end of 100 oscillations. When the oscillator completes 200 oscillations, its amplitude must be

(a) $a_0/2$ (b) $a_0/4$ (c) $a_0/6$ (d) $a_0/9$

22. The spring constant from the adjoining combination of springs is



(a) K
(b) 2K
(c) 4K
(d) 5K/2

23. A body executes simple harmonic motion. The potential energy (P.E), the kinetic energy (K.E) and total energy (T.E) are measured as a function of displacement x. Which of the following statements is true ?

(a) K.E. is maximum when $x = 0$
(b) T.E is zero when $x = 0$
(c) K.E is maximum when x is maximum
(d) P.E is maximum when $x = 0$

24. A simple harmonic wave having an amplitude a and time period T is represented by the equation $y = 5\sin\pi(t+4)m$. Then the value of amplitude (a) in (m) and time period (T) in second are

(a) a = 10, T = 2 (b) a = 5, T = 1
(c) a = 10, T = 1 (d) a = 5, T = 2

25. A particle moves such that its acceleration 'a' is given by a = – zx where x is the displacement from equilibrium position and z is constant. The period of oscillation is

(a) $2\pi/z$ (b) $2\pi/\sqrt{z}$ (c) $\sqrt{2\pi/z}$ (d) $2\sqrt{\pi/z}$

26. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is

(a) 0.25 s (b) 0.5 s (c) 0.75 s (d) 0.125 s

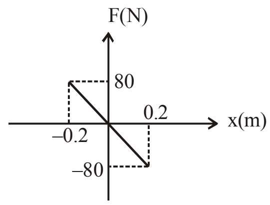
27. A tunnel has been dug through the centre of the earth and a ball is released in it. It executes S.H.M. with time period

(a) 42 minutes (b) 1 day
(c) 1 hour (d) 84.6 minutes

28. The displacement equation of a particle is $x = 3\sin 2t + 4\cos 2t$. The amplitude and maximum velocity will be respectively

(a) 5, 10 (b) 3, 2 (c) 4, 2 (d) 3, 4

29. A body of mass 0.01 kg executes simple harmonic motion about x = 0 under the influence of a force as shown in figure. The time period of SHM is



(a) 1.05 s
(b) 0.52 s
(c) 0.25 s
(d) 0.03 s

30. Two oscillators are started simultaneously in same phase. After 50 oscillations of one, they get out of phase by π, that is half oscillation. The percentage difference of frequencies of the two oscillators is nearest to

(a) 2% (b) 1% (c) 0.5% (d) 0.25%

31. The length of a second's pendulum at the surface of earth is 1 m. The length of second's pendulum at the surface of moon where g is 1/6th that at earth's surface is

(a) 1/6 m (b) 6 m (c) 1/36 m (d) 36 m

32. A simple spring has length l and force constant K. It is cut into two springs of lengths l_1 and l_2 such that $l_1 = n\, l_2$ (n = an integer). The force constant of spring of length l_1 is

(a) K(1 + n) (b) (K/n)(1 + n)
(c) K (d) K/(n + 1)

33. The displacement of a particle from its mean position (in metre) is given by $y = 0.2 \sin(10\pi t + 1.5\pi)\cos(10\pi t + 1.5\pi)$ The motion of particle is

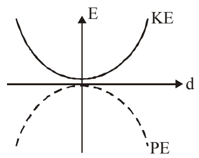
(a) periodic but not SHM
(b) non-periodic
(c) simple harmonic motion with period 0.1 s
(d) simple harmonic motion with period 0.2 s.

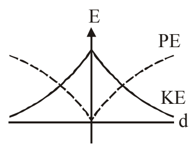
34. A point particle of mass 0.1 kg is executing S.H.M. of amplitude 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} joule. Obtain the equation of motion of this particle, if the initial phase of oscillation is 45°.

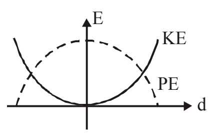
(a) $y = 0.1\sin\left(\pm 4t + \frac{\pi}{4}\right)$ (b) $y = 0.2\sin\left(\pm 4t + \frac{\pi}{4}\right)$

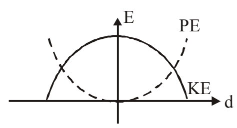
(c) $y = 0.1\sin\left(\pm 2t + \frac{\pi}{4}\right)$ (d) $y = 0.2\sin\left(\pm 2t + \frac{\pi}{4}\right)$

35. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (*graphs are schematic and not drawn to scale*)

(a)


(b)


(c)


(d)


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26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ
31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ

Space for Rough Work

36. The equation of a simple harmonic wave is given by

$$y = 3\sin\frac{\pi}{2}(50t - x)$$

Where x and y are in meters and t is in seconds. The ratio of maximum particle velocity to the wave velocity is

(a) 2π (b) $\frac{3}{2}\pi$ (c) 3π (d) $\frac{2}{3}\pi$

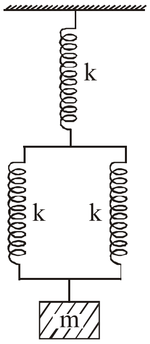
37. If the mass shown in figure is slightly displaced and then let go, then the system shall oscillate with a time period of

(a) $2\pi\sqrt{\frac{m}{3k}}$

(b) $2\pi\sqrt{\frac{3m}{2k}}$

(c) $2\pi\sqrt{\frac{2m}{3k}}$

(d) $2\pi\sqrt{\frac{3k}{m}}$



38. A hollow sphere is filled with water. It is hung by a long thread. As the water flows out of a hole at the bottom, the period of oscillation will

(a) first increase and then decrease
(b) first decrease and then increase
(c) go on increasing
(d) go on decreasing

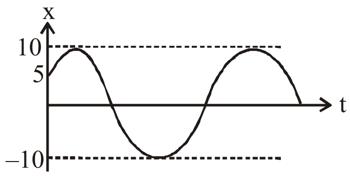
39. The figure shows a position time graph of a particle executing SHM. If the time period of SHM is 2 sec, then the equation of SHM is

(a) $x = 10\cos\pi t$

(b) $x = 5\sin\left(\pi t + \frac{\pi}{3}\right)$

(c) $x = 10\sin\left(\pi t + \frac{\pi}{3}\right)$

(d) $x = 10\sin\left(\pi t + \frac{\pi}{6}\right)$



40. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time

(a) at the mean position of the platform
(b) for an amplitude of $\frac{g}{\omega^2}$
(c) for an amplitude of $\frac{g^2}{\omega^2}$
(d) at the highest position of the platform

41. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m^3. The relationship between t and t_0 is

(a) $t = 2t_0$ (b) $t = t_0/2$ (c) $t = t_0$ (d) $t = 4t_0$

42. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy?

(a) $\frac{1}{6}$s (b) $\frac{1}{4}$s (c) $\frac{1}{3}$s (d) $\frac{1}{12}$s

43. A body executes simple harmonic motion under the action of a force F_1 with a time period $\frac{4}{5}$ s. If the force is changed to F_2, it executes S.H.M. with time period $\frac{3}{5}$ s. If both the forces F_1 and F_2 act simultaneously in the same direction on the body, its time period in second is

(a) $\frac{12}{25}$ (b) $\frac{7}{5}$ (c) $\frac{24}{25}$ (d) $\frac{5}{7}$

44. A block connected to a spring oscillates vertically. A damping force F_d, acts on the block by the surrounding medium. Given as $F_d = -bV$, b is a positive constant which depends on :

(a) viscosity of the medium
(b) size of the block
(c) shape of the block
(d) All of these

45. If a simple pendulum of length l has maximum angular displacement θ, then the maximum K.E. of bob of mass m is

(a) $\frac{1}{2}ml/g$ (b) $mg/2l$
(c) $mgl(1 - \cos\theta)$ (d) $mgl\sin\theta/2$

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	41. ⓐⓑⓒⓓ	42. ⓐⓑⓒⓓ	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ	45. ⓐⓑⓒⓓ

DAILY PRACTICE PROBLEM DPP CHAPTERWISE CP13 - PHYSICS			
Total Questions	45	Total Marks	180
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	45	Qualifying Score	60
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

Space for Rough Work

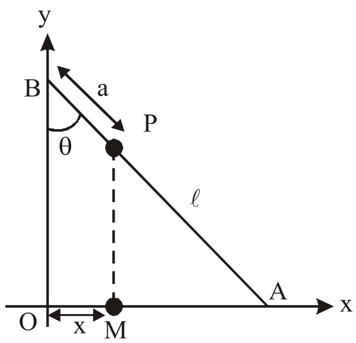
1. **(a)** For an SHM, the acceleration $a = -\omega^2 x$ where ω^2 is a constant. Therefore, $\frac{a}{x}$ is a constant. The time period T is also constant. Therefore, $\frac{aT}{x}$ is a constant.

2. **(b)** $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$ or $t_1^2 = \frac{4\pi^2 m}{k_1}$ or $k_1 = \frac{4\pi^2 m}{t_1^2}$

Similarly, $k_2 = \frac{4\pi^2 m}{t_2^2}$ and $(k_1 + k_2) = \frac{4\pi^2 m}{t_0^2}$

$\therefore \frac{4\pi^2 m}{t_0^2} = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$ or $\frac{1}{t_0^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$

3. **(b)**



$\frac{d\theta}{dt} = 2 \therefore \theta = 2t$

Let BP = a, $\therefore$ x = OM = a sin θ = a sin (2t)

Hence M executes SHM within the given time period and its acceleration is opposite to x that means towards left.

4. **(b)** The kinetic energy of a particle executing S.H.M. is given by

$K = \frac{1}{2} m a^2 \omega^2 \sin^2 \omega t$

where, m = mass of particle
a = amplitude
ω = angular frequency
t = time

Now, average K.E. $= \langle K \rangle = \langle \frac{1}{2} m\omega^2 a^2 \sin^2 \omega t \rangle$

$= \frac{1}{2} m\omega^2 a^2 \langle \sin^2 \omega t \rangle$

$= \frac{1}{2} m\omega^2 a^2 \left(\frac{1}{2}\right) \quad \left(\because \langle \sin^2\theta \rangle = \frac{1}{2}\right)$

$= \frac{1}{4} m\omega^2 a^2 = \frac{1}{4} ma^2 (2\pi\nu)^2 \quad (\because \omega = 2\pi\nu)$

or, $\langle K \rangle = \pi^2 m a^2 \nu^2$

5. **(a)** We know that $T = 2\pi\sqrt{\frac{M}{k}}$

From first case, $2 = 2\pi\sqrt{\frac{M}{k}}$ .... (1)

In second case, $4 = 2\pi\sqrt{\frac{M+2}{k}}$ .... (2)

From eq. (1) and eq. (2)

$\frac{4}{2} = \sqrt{\frac{M+2}{M}} \Rightarrow 4 = 1 + \frac{2}{M}$

$\frac{2}{M} = 3 \Rightarrow M = \frac{2}{3}$kg

6. **(b)** Amplitude of a damped oscillator at any instant t is given by

$A = A_0 e^{-bt/2m}$

where A_0 is the original amplitude

From question,

When t = 2 s, $A = \frac{A_0}{3}$

$\therefore \frac{A_0}{3} = A_0 e^{-2b/2m}$

or, $\frac{1}{3} = e^{-b/m}$ ...(i)

When t = 6 s, $A = \frac{A_0}{n}$

$\therefore \frac{A_0}{n} = A_0 e^{-6b/2m}$

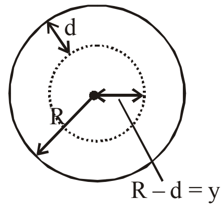
or, $\frac{1}{n} = e^{-3b/m} = (e^{-b/m})^3$

or, $\frac{1}{n} = \left(\frac{1}{3}\right)^3$ (Using eq. (i))

$\therefore n = 3^3$

7. **(c)** Acceleration due to gravity at a depth 'd' is given by,

$g' = g\left(1 - \frac{d}{R}\right) = g\left(\frac{R-d}{R}\right) = \frac{g}{R} y$



or acceleration ∝ displacement

which is the condition for SHM. So, body will oscillate simple harmonically in tunnel.

8. **(c)** Time to complete 1/4th oscillation is $\frac{T}{4}$ s. Time to complete $\frac{1}{8}$ th vibration from extreme position is obtained from

$$y = \frac{a}{2} = a\cos\omega t = a\cos\frac{2\pi}{T}t \text{ or } t = \frac{T}{6}s$$

So time to complete 3/8th oscillation

$$= \frac{T}{4} + \frac{T}{6} = \frac{5T}{12}$$

9. **(a)** As we know, kinetic energy $= \frac{1}{2}m\omega^2(A^2 - x^2)$

Potential energy $= \frac{1}{2}m\omega^2x^2$

$$\therefore \frac{\frac{1}{2}m\omega^2(A^2-x^2)}{\frac{1}{2}m\omega^2x^2} = \frac{1}{4} \Rightarrow \frac{A^2-x^2}{x^2} = \frac{1}{4}$$

$$4A^2 - 4x^2 = x^2 \Rightarrow x^2 = \frac{4}{5}A^2 \quad \therefore x = \frac{2}{\sqrt{5}}A.$$

10. **(d)** $T = 2\pi\sqrt{\frac{\ell}{g}}$ and $T' = 2\pi\sqrt{\frac{1.21\ell}{g}}$

$(\because \ell' = \ell + 21\% \text{ of } \ell)$

$$\% \text{ increase} = \frac{T' - T}{T} \times 100$$

$$= \frac{\sqrt{1.21\ell} - \sqrt{\ell}}{\sqrt{\ell}} \times 100 = \left(\sqrt{1.21} - \sqrt{1}\right) \times 100$$

$$= (1.1 - 1) \times 100 = 10\%$$

11. **(d)** $T = 2\pi\sqrt{\frac{m}{k}}$

When a spring is cut into n parts

Spring constant for each part $= nk$

Here, $n = 4$

$$T_1 = 2\pi\sqrt{\frac{m}{4k}} = \frac{T}{2}$$

12. **(d)** $x = A\cos(\omega t + \delta)$

$$y = A\cos(\omega t + \alpha) \quad ....(1)$$

When $\delta = \alpha + \frac{\pi}{2}$

$$x = A\cos\left(\frac{\pi}{2} + \omega t + \alpha\right)$$

$$x = -A\sin(\omega t + \alpha) \quad ....(2)$$

Squaring (1) and (2) and then adding

$x^2 + y^2 = A^2[\cos^2(\omega t + \alpha) + \sin^2(\omega t + \alpha)]$

or $x^2 + y^2 = A^2$, which is the equation of a circle. The present motion is anticlockwise.

13. **(a)** Here,

$x = x_0\cos(\omega t - \pi/4)$

$\therefore$ Velocity, $v = \frac{dx}{dt} = -x_0\omega\sin\left(\omega t - \frac{\pi}{4}\right)$

Acceleration,

$$a = \frac{dv}{dt} = -x_0\omega^2\cos\left(\omega t - \frac{\pi}{4}\right)$$

$$= x_0\omega^2\cos\left[\pi + \left(\omega t - \frac{\pi}{4}\right)\right]$$

$$= x_0\omega^2\cos\left(\omega t + \frac{3\pi}{4}\right) \quad ...(1)$$

Acceleration, $a = A\cos(\omega t + \delta)$ ...(2)

Comparing the two equations, we get

$A = x_0\omega^2$ and $\delta = \frac{3\pi}{4}$.

14. **(c)** $T = 2\pi\sqrt{\frac{M}{k}}$

$$T' = 2\pi\sqrt{\frac{M+m}{k}} = \frac{5T}{3}$$

$$\therefore 2\pi\sqrt{\frac{M+m}{k}} = \frac{5}{3} \times 2\pi\sqrt{\frac{M}{k}}$$

$$M + m = \frac{25}{9} \times M$$

$$1 + \frac{m}{M} = \frac{25}{9} \Rightarrow \frac{m}{M} = \frac{25}{9} - 1 = \frac{16}{9}$$

15. **(d)** $A = 0.05$ m, $y = 0.01$ m

Acceleration, $a = 1.0\ m/s^2$

We have, $a = -\omega^2 y$ or $|a| = \omega^2 y$

$$\Rightarrow 1.0 = \omega^2 \times 0.01$$

$$\therefore \omega^2 = \frac{1.0}{0.01} = 100 \Rightarrow \omega = 10$$

Now, time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{10} = \frac{\pi}{5}$ sec.

16. **(c)** We have, $U + K = E$

where, U = potential energy, K = Kinetic energy, E = Total energy.

Also, we know that, in S.H.M., when potential energy is maximum, K.E. is zero and vice-versa.

$\therefore U_{max} + 0 = E \Rightarrow U_{max} = E$

Further,

$$K.E. = \frac{1}{2}m\omega^2a^2\cos^2\omega t$$

But by question, $K.E. = K_0\cos^2\omega t$

$$\therefore K_0 = \frac{1}{2}m\omega^2a^2$$

Hence, total energy, $E = \frac{1}{2}m\omega^2a^2 = K_0$

$\therefore U_{max} = K_0$ & $E = K_0$.

17. (b) Distance covered by lift is given by
$y = t^2$
∴ Acceleration of lift upwards

$$= \frac{d^2y}{dt^2} = \frac{d}{dt}(2t) = 2\ m/s^2 = \frac{g}{5}$$

Now, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$T' = 2\pi\sqrt{\frac{\ell}{g+\frac{g}{5}}} = 2\pi\sqrt{\frac{\ell}{\frac{6}{5}g}} = \sqrt{\frac{5}{6}}T.$$

18. (d) In simple harmonic motion, starting from rest,
At $t = 0$, $x = A$
$x = A\cos\omega t$ ..... (i)
When $t = \tau$, $x = A - a$
When $t = 2\tau$, $x = A - 3a$
From equation (i)
$A - a = A\cos\omega\tau$ ......(ii)
$A - 3a = A\cos 2\omega\tau$ ....(iii)
As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$ ...(iv)
From equation (ii), (iii) and (iv)

$$\frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

$$\Rightarrow \frac{A-3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$\Rightarrow A^2 - 3aA = A^2 + 2a^2 - 4Aa$
$\Rightarrow 2a^2 = aA$
$\Rightarrow A = 2a$

$$\Rightarrow \frac{a}{A} = \frac{1}{2}$$

Now, $A - a = A\cos\omega\tau$

$$\Rightarrow \cos\omega\tau = \frac{A-a}{A}$$

$$\Rightarrow \cos\omega\tau = \frac{1}{2} \quad \text{or} \quad \frac{2\pi}{T}\tau = \frac{\pi}{3}$$

$\Rightarrow T = 6\tau$

19. (b) $v_1 = \frac{dy_1}{dt} = 0.1 \times 100\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$

$$v_2 = \frac{dy_2}{dt} = -0.1\pi\sin\pi t = 0.1\pi\cos\left(\pi t + \frac{\pi}{2}\right)$$

$$\therefore \text{Phase diff.} = \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6} = -\frac{\pi}{6}$$

20. (c) $f_A = \frac{1}{2\pi}\sqrt{\frac{g}{L_A}}$ and $f_B = \frac{f_A}{2} = \frac{1}{2\pi}\sqrt{\frac{g}{L_B}}$

$$\therefore \frac{f_A}{f_{A/2}} = \frac{1}{2\pi}\sqrt{\frac{g}{L_A}} \times 2\pi\sqrt{\frac{L_B}{g}} \Rightarrow 2 = \sqrt{\frac{L_B}{L_A}} \Rightarrow 4 = \frac{L_B}{L_A},$$

regardless of mass

21. (d)

22. (c) Here all the three springs are connected in parallel to mass m. Hence equivalent spring constant $k = K + K + 2K = 4K$.

23. (a) $K.E. = \frac{1}{2}m\omega^2(a^2 - x^2)$

When $x = 0$, K.E is maximum and is equal to $\frac{1}{2}m\omega^2 a^2$.

24. (d) $y = 5\sin(\pi t + 4\pi)$, comparing it with standard equation

$$y = a\sin(\omega t + \phi) = a\sin\left(\frac{2\pi t}{T} + \phi\right)$$

$a = 5m$ and $\frac{2\pi t}{T} = \pi t \Rightarrow T = 2$ sec.

25. (b) $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{x}{zx}} = 2\pi/\sqrt{z}$

26. (b) Here, $x = 2 \times 10^{-2}\cos\pi t$
Speed is given by

$$v = \frac{dx}{dt} = 2 \times 10^{-2}\ \pi\sin\pi t$$

For the first time, the speed to be maximum,
$\sin\pi t = 1$ or, $\sin\pi t = \sin\frac{\pi}{2}$

$$\Rightarrow \pi t = \frac{\pi}{2} \text{ or, } t = \frac{1}{2} = 0.5 \text{ sec.}$$

27. (d) $T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{64 \times 10^6}{9.8}} = 2 \times \frac{22}{7} \times \frac{8 \times 10^3}{7 \times \sqrt{2}}$

$$= \frac{\sqrt{2} \times 22 \times 8 \times 1000}{49 \times 60} \text{ min} = 84.6 \text{ min}$$

28. (a) $x = 3\sin 2t + 4\cos 2t$. From given equation

$a_1 = 3$, $a_2 = 4$, and $\phi = \frac{\pi}{2}$

$$\therefore a = \sqrt{a_1^2 + a_2^2} = \sqrt{3^2 + 4^2} = 5$$

$\Rightarrow v_{max} = a\omega = 5 \times 2 = 10$

29. (d) Slope of F - x curve $= -k = -\frac{80}{0.2} \Rightarrow k = 400$ N/m,

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 0.0314$ sec.

30. (b) Phase change π in 50 oscillations.
Phase change 2π in 100 oscillations.
So frequency different ~ 1 in 100.

31. (a) $T = 2\pi\sqrt{\frac{\ell}{g}}$; $2 = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell'}{(g/6)}}$
Time period will remain constant if on moon,
$\ell' = \ell/6 = 1/6$ m

32. (b) Let k be the force constant of spring of length l_2. Since $l_1 = n\,l_2$, where n is an integer, so the spring is made of $(n + 1)$ equal parts in length each of length l_2.

$\therefore \quad \frac{1}{K} = \frac{(n+1)}{k}$ or $k = (n+1)K$

The spring of length l_1 (= n l_2) will be equivalent to n springs connected in series where spring constant

$k' = \frac{k}{n} = (n+1)K/n$ & spring constant of length ℓ_2 is K(n+1).

33. (c) Given

$y = 0.2 \sin(10\pi t + 1.5\pi)\cos(10\pi t + 1.5\pi)$

We know that 2 sinA cos A = sin2A, we get

$y = 0.1 \sin 2\ (10\pi t + 1.5\pi) = 0.1 \sin\ (20\pi t + 3\pi)$

On comparing with wave equation

$y = a \sin(\omega t + \phi)$ we get

$\omega = 20\pi$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = \frac{1}{10}\text{sec.} = 0.1\text{sec.}$

34. (a) The displacement of a particle in S.H.M. is given by

$y = a \sin(\omega t + \phi)$

velocity $= \frac{dy}{dt} = \omega a \cos(\omega t + \phi)$

The velocity is maximum when the particle passes through the mean position i.e.,

$$\left(\frac{dy}{dt}\right)_{max} = \omega a$$

The kinetic energy at this instant is given by

$$\frac{1}{2}m\left(\frac{dy}{dt}\right)^2_{max} = \frac{1}{2}m\omega^2 a^2 = 8 \times 10^{-3}\text{ joule}$$

or $\quad \frac{1}{2} \times (0.1)\,\omega^2 \times (0.1)^2 = 8 \times 10^{-3}$

Solving we get $\quad \omega = \pm 4$

Substituting the values of a, ω and ϕ in the equation of S.H.M., we get

$y = 0.1 \sin(\pm 4t + \pi/4)$ metre.

35. (d) $K.E = \frac{1}{2}k(A^2 - d^2)$

and $P.E. = \frac{1}{2}kd^2$

At mean position $d = 0$. At extrement positions $d = A$

36. (b) $y = 3\sin\frac{\pi}{2}(50t - x)$

$y = 3\sin\left(25\pi t - \frac{\pi}{2}x\right)$ on comparing with the standard wave equation

$y = a\sin(\omega t - kx)$

Wave velocity $v = \frac{\omega}{k} = \frac{25\pi}{\pi/2} = 50$ m/sec.

The velocity of particle

$$v_p = \frac{\partial y}{\partial t} = 75\pi\cos\left(25\pi t - \frac{\pi}{2}x\right)$$

$v_{p\,max} = 75\pi$

then $\frac{v_{p\max}}{v} = \frac{75\pi}{50} = \frac{3\pi}{2}$

37. (b) The equivalent situation is a series combination of two springs of spring constants k and 2k.

If k' is the equivalent spring constant, then

$$k' = \frac{(k)(2k)}{3k} = \frac{2k}{3}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{3m}{2k}}$$

38. (a) Time period of simple pendulum $T = 2\pi\sqrt{\left(\frac{l}{g}\right)} \propto \sqrt{l}$

where l is effective length.

[i.e distance between centre of suspension and centre of gravity of bob]

Initially, centre of gravity is at the centre of sphere. When water leaks the centre of gravity goes down until it is half filled; then it begins to go up and finally it again goes at the centre. That is effective length first increases and then decreases. As $T \propto \sqrt{l}$, so time period first increases and then decreases.

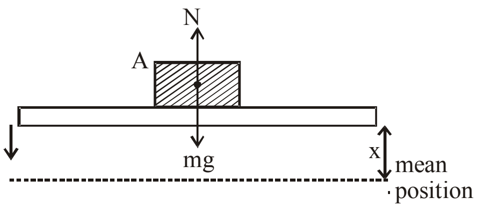
39. (d) At $t = 0$, $x = 5 = \frac{A}{2}$

$\Rightarrow$ Initial phase, $\phi = 30° = \frac{\pi}{6}$

$\Rightarrow x = A\sin(\omega t + \phi)$

$= 10\sin\left(\frac{2\pi}{T}t + \frac{\pi}{6}\right) = 10\sin\left(\pi t + \frac{\pi}{6}\right)$

40. (b) For block A to move in S.H.M.



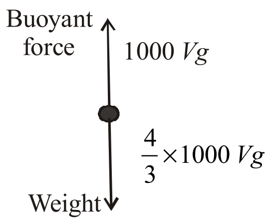
$mg - N = m\omega^2 x$

where x is the distance from mean position

For block to leave contact N = 0

$\Rightarrow mg = m\omega^2 x \Rightarrow x = \frac{g}{\omega^2}$

41. (a) $t = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$; $t_0 = 2\pi\sqrt{\frac{\ell}{g}}$



Net force $= \left(\frac{4}{3}-1\right)\times 1000\ Vg = \frac{1000}{3}Vg$

$$g_{eff} = \frac{1000\ Vg}{3\times\frac{4}{3}\times 1000V} = \frac{g}{4}$$

$$\therefore t = 2\pi\sqrt{\frac{\ell}{g/4}}$$

$t = 2t_0$

42. (a) K.E. of a body undergoing SHM is given by,

$$K.E. = \frac{1}{2}ma^2\omega^2\cos^2\omega t$$

$$T.E. = \frac{1}{2}ma^2\omega^2$$

Given K.E. = 0.75 T.E.

$$\Rightarrow 0.75 = \cos^2\omega t \Rightarrow \omega t = \frac{\pi}{6}$$

$$\Rightarrow t = \frac{\pi}{6\times\omega} \Rightarrow t = \frac{\pi\times 2}{6\times 2\pi} \Rightarrow t = \frac{1}{6}\ s$$

43. (a) Under the action of first force, $F_1 = m\omega_1^2 y$

Under the action of second force,

$F_2 = m\omega_2^2 y$

Under the action of resultant force,

$F_1 + F_2 = m\omega^2 y$

$\Rightarrow m\omega^2 y = m\omega_1^2 y + m\omega_2^2 y$

$\Rightarrow \omega^2 = \omega_1^2 + \omega_2^2$

$$\Rightarrow \left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{T_1}\right)^2 + \left(\frac{2\pi}{T_2}\right)^2$$

$$\Rightarrow T = \sqrt{\frac{T_1^2T_2^2}{T_1^2+T_2^2}} = \sqrt{\frac{\left(\frac{4}{5}\right)^2\cdot\left(\frac{3}{5}\right)^2}{\left(\frac{4}{5}\right)^2+\left(\frac{3}{5}\right)^2}} = \frac{12}{25}.$$

44. (d) $F = -bV$, b depends on all the three i.e, shape and size of he block and viscosity of the medium.

45. (c) When the bob moves from maximum angular displacement θ to mean position, then the loss of gravitational potential energy is mgh

where $h = l(1-\cos\theta)$